THE ASSESSMENT OF FORECAST INTERVALS UNCERTAINTY FOR OIL PRICES

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Abstract:
The main objective of this study is to assess the uncertainty of daily forecast intervals for highs and lows of WTI crude oil spot prices. For constructing the prediction intervals on the horizon 24th of February 2014-25th of March 2014, different quantitative methods were used, the historical errors method providing the best results. All the tests (independence test, the unconditional coverage and the combined test) conduct us to the same result: only for the forecast intervals based on historical error method there are not significant differences based on ex-ante and ex-post probability.

Key words: *forecast interval, uncertainty, LR tests, historical errors method, oil prices*

1. Introduction

The forecasts uncertainty is the main cause of actual economic crisis. The common current in literature that explains a world economic crisis registered a major failure. Authors like Novy and Taylor (2012) demonstrated that the predictions uncertainty is the real cause of the commercial collapse from US during 2008-2009. Therefore, the construction of forecast intervals is a better solution, but these intervals should be accompanied by a proper assessment of uncertainty.

Authors like Bachmann, Elstner and Sims (2010) and Bloom and Davis (2013) showed that the policy uncertainty is the real cause of the decrease in actives profit. Leduc and Liu (2012) observed that the actual economic crisis has as important characteristic the higher uncertainty that diminished more the economic activity compared to the previous crisis.

Three usual uncertainty measures were utilized by Giordani and Söderlind (2003): standard deviation of individual predictions, disagreement between forecasters, and the variance of aggregated histogram. According to Ericsson (2001) the most used statistical indicators for forecasts' uncertainty are:

1. The bias of the prediction;
2. The variance of forecast error;
3. Mean Square Error (MSE).

The forecast intervals are based on the point predictions, forecast error and a probability that is associated according to the assumption referring to the errors repartition. In the general case it is made the assumption that the random shocks have a normal distribution $e_t \to N(0,\sigma_\varepsilon^2)$ that supposes a normally distributed probability density $x_{t+h} \to N(\hat{x}_{t+h},\sigma_h^2)$.

From the very beginning the experts used point predictions for past periods in order to have a proxy as an uncertainty measure. These indicators are compared to ex-ante uncertainty measures.

Wallis (2008) considered that the consensus is the agreement degree related to the point forecasts made by specialists for a certain variable. The authors defined the uncertainty as variance of probability distributions.

The main objective of this article is to construct and assess the uncertainty of the forecast intervals for oil prices. Therefore, the research is structured as it follows: after a short introduction, the second section presents the methods for building the prediction intervals. Then, the tests for assessing the forecast intervals are described and the results of evaluation for real data are made.

2. The construction of the forecast intervals

The variable at moment t is denoted by x_t and it is actually an observation of a random variable (X_t). A random walk (first-order autoregressive model- AR(1)) is written as:

$$X_t = \alpha X_{t-1} + \varepsilon_t \qquad (1)$$

α- constant (for stationary data series $|\alpha| < 1$)

ε_t- error at moment t

A model with additive errors is written as:

$$X_t = \mu_t + \varepsilon_t \qquad (2)$$

μ_t- predictable component of the model

$\{\varepsilon_t\}$- sequence of independent normally distributed random variables (null average and constant dispersion: NID($0,\sigma_\varepsilon^2$)

The exponential smoothing calculates a point prediction by creating a weighted average of the latest observation and the most recent point prediction. It is an optimal method (it has the least mean squared error predictions) for the following model (ARIMA(0,1,1):

$$X_t = X_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1} \quad (3)$$

The 100(1- α)% prediction interval (P.I.) for the h steps ahead forecasts are computed as:

$$\hat{x}_n(h) \mp z_{\alpha/2}\sqrt{Var(e_n(h))} \quad (4)$$

$z_{\alpha/2}$ –two-tailed percentage point of the normal distribution of null mean and dispersion equaled to 1

This PI is symmetric about $\hat{x}_n(h)$, the point prediction being unbiased. The uncertainty in predictions for only one variable is assessed using expected means square prediction error (PMSE). If we make the comparison between predictions for different variables, it is recommended the use of MAPE (mean absolute prediction error).

Exponential smoothing method can be utilized for data set with no obvious tendency or seasonality. The PMSE in this case is given by:

$$Var(e_n(h)) = [1 + (h-1)\alpha^2]\sigma_\varepsilon^2 \quad (5)$$

α- smoothing parameter

$Var(e_n(h))$- variance of the one-step-ahead prediction errors

For the random walk, the variance of predictions error is computed as:

$$Var(e_n(h)) = h\,\sigma_\varepsilon^2 \qquad (6)$$

Prediction intervals work under the assumption that the prediction errors follow a normal repartition of zero mean and a standard deviation represented by the indicator called root mean square error (RMSE) based on historical prediction errors. If the probability is (1-α), the prediction interval is determined as:

$$(X_t(k) - z_{\alpha/2} \cdot RMSE(k),\ X_t(k) + z_{\alpha/2} \cdot RMSE(k)), k = 1,...,K \qquad (7)$$

$X_t(k)$- point prediction of variable X_{t+k} made at the moment t

$z_{\alpha/2}$- the α/2-th quintile of standardized normal repartition.

3. The evaluation of forecast intervals for oil price

The data used in this paper are the observed daily highs (op1) and lows (op2) of WTI crude oil spot prices. These data cover many years, the first observation being 1st January 1986 and the last being 24th of February 2014. The data are expressed in log scale, being utilized to construct the forecast model on the horizon from 25th of February to 25th of March. An ex-post assessment was made for the period till 11th of March. For building the models, the data are stationarized by differentiating them.
The models used in making predictions have the following form:

$$dop1_t = -0.0373 \cdot op_{t-1} + \varepsilon_t \quad (8)$$

$$dop2_t = 0.1473 \cdot dop2_{t-1} + \varepsilon_t + 0.1808 \cdot \varepsilon_{t-1} \quad (9)$$

Table 1: Point forecasts and prediction intervals based on econometric models for the next month (25 th February 2014- 25th of March 2014)

Day	op1	Lower limit op1	Upper limit op1	op2	Lower limit op2	Upper limit op2	Actual values op1	Actual values op2
2014-February 25	102.14	101.88	103.40	106.81	104.75	109.97	**103.17**	**109.76**
2014-February 26	101.12	100.75	102.49	103.14	100.84	105.84	**102.2**	109.19
2014-February 27	100.11	99.66	100.56	101.13	98.31	105.17	102.93	109.39
2014-February 28	99.10	98.57	99.63	100.84	97.12	107.92	102.68	108.54
2014-March 3	96.16	95.57	96.75	99.83	94.56	103.98	102.88	108.98
2014-March 4	95.20	94.56	95.84	98.45	90.55	105.30	105.34	111.26
2014-March 5	94.25	93.56	94.94	97.34	84.85	106.96	103.64	109.17
2014-March 6	93.31	92.57	94.05	96.57	85.85	107.51	101.75	108.15
2014-March 7	92.37	91.58	93.16	95.8	89.92	102.63	101.82	107.99
2014-March 10	89.63	88.80	90.46	95.03	90.35	103.73	102.82	109.14
2014-March 11	88.73	87.86	89.60	94.26	96.75	107.70	101.39	108.27
2014-March 12	87.84	86.93	88.75	93.49	90.85	108.39		
2014-	86.97	86.02	87.92	92.72	90.96	112.30		

March 13								
2014- March 14	86.10	85.12	87.08	91.95	90.26	107.18		
2014- March 17	83.54	82.52	84.56	91.18	89.45	110.19		
2014- March 18	82.70	81.65	83.75	90.41	87.67	101.22		
2014- March 19	81.88	80.80	82.96	89.64	86.04	103.69		
2014- March 20	81.06	79.95	82.17	88.87	85.08	104.64		
2014- March 21	80.25	79.11	81.39	88.1	86.56	102.84		
2014- March 24	77.86	76.69	79.03	87.33	86.99	103.58		
2014- March 25	77.09	75.89	78.29	86.56	80.69	105.03		

Source: author's computations

At first glance, the results showed that only two values of op1 were located in the PI and only one actual value of the op2 was situated in the PI. Therefore, the historical errors method is applied for the same variables.

Table 2: Point forecasts and prediction intervals based on historical errors method for the next month (25 th February 2014- 25th of March 2014)

Day	Lower limit op1	Upper limit op1	Lower limit op2	Upper limit op2	Actual values op1	Actual values op2
2014- February 25	101.95	104.03	105.56	110.37	**103.17**	**109.76**
2014- February 26	101.28	103.82	106.12	110.93	**102.2**	**109.19**
2014- February 27	100.38	102.98	106.51	110.02	**102.93**	**109.39**
2014- February 28	101.37	103.23	106.85	110.33	**102.68**	**108.54**
2014- March 3	102.55	103.82	106.91	110.62	**102.88**	**108.98**
2014- March 4	102.97	103.91	107.33	110.34	105.34	111.26
2014- March 5	102.57	104.03	107.49	110.62	**103.64**	**109.17**
2014- March 6	102.69	104.16	107.77	111.21	101.75	**108.15**
2014- March 7	102.88	104.25	107.83	111.08	101.82	**107.99**
2014- March 10	102.74	104.41	107.55	111.36	**102.82**	**109.14**
2014- March 11	102.63	104.53	108.33	111.98	101.39	108.27
2014- March 12	102.76	104.59	108.34	112.09		
2014- March 13	102.93	104.67	108.29	112.53		
2014- March 14	103.02	104.78	108.52	112.83		

2014- March 17	103.22	105.93	108.72	112.92		
2014- March 18	103.31	105.97	108.77	113.15		
2014- March 19	103.45	106.42	108.89	113.91		
2014- March 20	103.56	105.76	108.91	113.97		
2014- March 21	103.67	106.83	109.62	114.04		
2014- March 24	103.72	106.92	109.73	114.28		
2014- March 25	103.81	107.23	109.88	114.56		

Source: author's computations

For assessing the uncertainty of forecast intervals more tests are used.

I. Likelihood ratio (LR) tests

For the test of unconditional coverage, the time series are used to make forecast intervals and the ex-ante probability is π expressing the probability that a value be in that interval. The objective is the assessment of ex-ante probability correction. n_1 values are in the forecast intervals and n_2 outside of these intervals. The coverage probability is defined as: p=n_1/n. For a binomial repartition, for the null hypothesis, the likelihood is: $L(\pi)=(1-\pi)^{n_2}\,\pi^{n_1}$. Under the alternative hypothesis the likelihood is: $L(p)=(1-p)^{n_2}\,p^{n_1}$. According to Christoffersen (1998) the statistic of likelihood ratio is: $LR_{UC}=2(n_2\log\frac{1-p}{1-\pi}+n_1\log\frac{p}{\pi})\xrightarrow[H_0]{}\chi_1^2$.

The independence test uses a matrix of transition frequencies $[n_{ij}]$ that is the number of observations in state i at time t-1 and in state j at time t. The maximal likelihood estimations of transition probabilities are calculated as the frequencies in a cell and the total number of frequencies of a line. Two cases are possible: the values are in the interval or outside. For these cases we used the values 1 and 0. The transition matrix of estimated probabilities is determined as:

$$P=\left\langle\begin{matrix}1-p_{01} & p_{01}\\ 1-p_{11} & p_{11}\end{matrix}\right\rangle=\left\langle\begin{matrix}\frac{n_{00}}{n_{0.}} & \frac{n_{01}}{n_{0}.}\\ \frac{n_{10}}{n_{1.}} & \frac{n_{11}}{n_{1.}}\end{matrix}\right\rangle$$

The likelihood of P is: $L(P)=(1-p_{01})^{n_{00}}\cdot p_{01}^{n_{01}}\cdot(1-p_{11})^{n_{10}}\cdot p_{11}^{n_{11}}$. The null hypothesis (H0) states that there is independence between the two states: t-1 and t. This assumption implies $\pi_{01}=\pi_{11}$. The estimator of maximal likelihood of the common probability is: $p=\frac{n_{.1}}{n}$. The likelihood under H0 evaluated at p is: $L(p)=(1-p)^{n_{.0}}\cdot p^{n_{1.}}$. The LR test statistic is: $LR_{ind}=-2\log\frac{L(p)}{L(P)}\xrightarrow[H_0]{}\chi_1^2$.

Christoffersen (1998) also proposed a test that combines the independence test and the unconditional coverage, the statistic of this test being: $LR_{CC}=-2\log\frac{L(\pi)}{L(P)}\xrightarrow[H_0]{}\chi_2^2$.

If the first observation is not taken into consideration, then: $LR_{CC}=LR_{UC}+LR_{ind}$.

II. Chi-square (χ^2) tests

Stuart, Ord and Arnold (1999) showed that likelihood ratio tests provide the same result as Pearson's goodness-of-fit tests. The statistic of chi-square test for unconditional coverage is: $X^2=\frac{n(p-\pi)^2}{\pi(1-\pi)}$ $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. The matrix of observed frequencies is $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

$$X^2=\frac{n(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)}$$

The conditional coverage test combined with the independence test uses the contingency table of the observed frequencies with expected frequencies under the null hypothesis of independent lines and using the coverage probability.

The matrix of expected frequencies is: $\begin{bmatrix} (1-\pi)(a+b) & \pi(a+b) \\ (1-\pi)(c+d) & \pi(c+d) \end{bmatrix}$. The statistic is the sum of square normal standard statistics of the normal samples proportions.

The cover probability is 2/11 for op1 and 1/11 for op2 in the case of intervals based on econometric models. The cover probability is greater for the intervals based on historical errors method: 7/11 for op1 and 9/11 for op2.

Table 3: The statistics of LR tests

Statistic	op1 (AR model)	op2 (ARMA model)	op1(historical errors method)	op2 (historical errors method)
LR_{UC}	3.97	4.45	0.37	0.16
LR_{ind}	7.23	6.87	4.64	4.33
LR_{CC}	6.76	6.55	4.37	4.22

Source: author's computations

The critical value for one degree of freedom and a significance level of 0.05 for unconditional test is 3.841, which is higher than the computed value only for the forecast intervals based on historical errors method. So, there are not significant differences between the ex-ante probability and the real probability at a 5% significance level only for the forecast intervals based on historical errors method.

For the independence test the computed value is compared to the critical one of 5.991. The same conclusion was obtained: there are not significant differences between the ex-ante and the ex-post probability for intervals based on historical errors method.

4. Conclusions

For daily oil price some forecast intervals are constructed for a month and these intervals are assessed using some statistical tests. All the tests (independence test, the unconditional coverage and the combined test) conduct us to the same result: only for the forecast intervals based on historical error method there are not significant differences based on ex-ante and ex-post probabilities.

5. References

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