VALUES OF LAND AND RENEWABLE RESOURCES IN A THREE-SECTOR ECONOMIC GROWTH MODEL

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Abstract:

This paper studies dynamic interdependence of capital, land and resource values in a three sector growth model with endogenous wealth and renewable resources. The model is based on the neoclassical growth theory, Ricardian theory and growth theory with renewable resources. The household's decision is modeled with an alternative approach proposed by Zhang two decades ago. The economic system consists of the households, industrial, agricultural, and resource sectors. The model describes a dynamic interdependence between wealth accumulation, resource change, and division of labor under perfect competition. We simulate the model to demonstrate the existence of a unique stable equilibrium point and plot the motion of the dynamic system. The study conducts comparative dynamic analysis with regard to changes in the propensity to consume resources, the propensity to consume housing, the propensity to consume agricultural goods, the propensity to consume industrial goods, the propensity to save, the population, and the output elasticity of capital of the resource sector.

Key words: *land value and rent, economic growth, economic structure, stock of renewable resources, price of renewable resources*

1. Introduction

Different kinds of capital, such as physical capital, human capital, resources, environment, and infrastructures, play varying role in economic growth and development. As these capitals vary over time due to depreciation, consuming and accumulation by people's efforts, it is significant to study how these capitals and their exchange values change over time. Nevertheless, as the history of economic analysis shows, it is quite difficult to build genuine dynamic models with interactions between multiple kinds of capitals within a compact analytical framework. The purpose of this study is to make a contribution to the literature of economic dynamics by developing a dynamic interdependence between capital, land and resource values in a three sector growth model with endogenous wealth and renewable resources. The model is based on the neoclassical growth theory, Ricardian theory and growth theory with renewable resources. The household's decision is modeled with an alternative approach proposed by Zhang two decades ago. The economic system consists of the households, industrial, agricultural, and resource sectors.

As far as physical capital and wealth accumulation are concerned, the model in this study is based on the neoclassical growth theory. Most of the models in the neoclas¬sical growth theory are extensions and generalizations of the pioneering works of Solow in 1956. The model has played an important role in the development of economic growth theory by using the neoclassical production function and neoclassical production theory. The Solow model has been extended and generalized in numerous directions (e.g., Uzawa, 1961; Kurz, 1963; Diamond, 1965; Stiglitz, 1967; Drugeon and Venditti, 2001; Erceg et al. 2005). Nevertheless, economic growth theory still lacks profound formal economic models for explaining relations between economic growth, economic structure, resources, and land value. One of the main reasons for the lacking of examining these interactions within a compact framework is that the problems should be dealt with within a dynamic framework with rational behavior mechanisms. Nevertheless, the three main frameworks in modeling household behavior in economic growth theory with capital accumulation are not proper for studying economic problems with high heterogeneity. The Solow model is the starting point for almost all analyses of economic growth (Solow, 1956). Household's wealth accumulation of the Solow model is not based on a mechanism of endogenous savings. Another important approach is the so-called representative agent growth model based Ramsey's utility function (Ramsey, 1928). Cass and Koopmans combined Ramsey's analysis of consumer optimization and Solow's description of profit-maximizing producers within a compact framework (Cass, 1965; Koopmans, 1965). Irrespective of many efforts in applying the Ramsey approach, it has become evident that the approach is not effective for dealing with economic problems with high heterogeneity. Another approach in economic modeling is the so-called OLG approach. In his original contribution to growth theory with capital accumulation, Diamond (1965) used the overlapping generations structure to examine the long-term dynamical efficiency of competitive production economies. The model has become a standard tool in macroeconomics to study economic dynamics in discrete time. Many growth models of macroeconomics are built within the OLG framework. The approach is a discrete version of the continuous Ramsey approach. Most models of the approach assume that agents live only two periods – as mentioned in Azariadis (1993), each period should last over 30 years if one really wants to use analytical results to provide direct insights into reality. The length of over 30 years period is generally considered too long for discussing modern economic changes because within each period nothing is allowed to be changeable. This study will model behavior of households with an alternative approach proposed by Zhang in the early 1990s (Zhang, 1993). The approach overcomes the lacking of microfoundation for household behavior in the Solow model and avoids the problems in the Ramsey approach.

This study is concerned with dynamic relations of renewable resources with capital accumulation and land-use patterns. Stock of renewable resources is changeable according how fast agents utilize resources and how fast renewable resources grow. We integrate the Solow one-sector growth, Uzawa-Lucas two-sector and some neoclassical growth models with renewable resource models. It is well known that natural resources are incorporated into the neoclassical growth theory in the 1970s (e.g., Plourde, 1970, 1971; Stiglitz, 1974; Clark, 1976; Dasgupta and Heal, 1979). Economists were aware of the necessity of modeling resources with dynamic theory long before. As early as in 1956 Gordon (1956) emphasized the need for a dynamic approach to fisheries economics: “The conservation problem is essentially one which requires a dynamic formulation… The economic justification of conservation is the same as that of any capital investment – by postponing utilization we hope to increase the quantity available for use at a future date. In the fishing industry we may allow our fish to grow and to reproduce so that the stock at a future date will be greater than it would be if we attempted to catch as much as possible at the present time. … [I]t is necessary to arrive at an optimum which is a catch per unit of time, and one must reach this objective through consideration of the interaction between the rate of catch, the dynamics of fish population, and the economic time-preference schedule of the community or the interest rate on invested capital. This is a very complicated problem and I suspect that we will have to look to the mathematical economists for assistance in clarifying it.” As pointed out by Munro and Scott (1985), in the 1950s it was quite difficult to develop workable dynamic models of resources. Solow (1999) also argues for the necessity of taking account of natural resources in the neoclassical growth theory. According to Solow if the resource good is used as one of the inputs in the production, then it is easy to incorporate the use of renewable resources into the neoclassical growth model. Nevertheless, Solow does not show how to incorporate possible consumption of renewable resource into the growth model. There are only a few models of growth and renewable resources which treat the renewable resource as a source of utility (see, Beltratti, et al., 1994, Ayong Le Kama, 2001). Our model contains the renewable resource as a source of utility. It should be noted that there are also studies on dynamic interactions among economic growth, renewable resources and elastic labor supply on the basis of the neoclassical growth theory with capital accumulation and renewable resource (e.g., Eliasson and Turnovsky, 2004, Alvarez-Cuadrado and van Long, 2011). Our model differs from these studies not only in that we use an alternative utility function, but also in that we introduce land into the growth theory with capital and resource.

As pointed out by Gaffney, M. (2008: 119), “Most economists today live in a two-factor world: There is just labor and capital. Land, so central to classical political economy, has been swallowed into capital and "disappeared."” Determination of land values and dynamics of land values are important in contemporary market economies. Common households may accumulate wealth by owning land and other kinds of wealth. It is obvious that determination of land values involves taking account of nonlinear dynamic interactions among many variables. It may explain why economics still lacks profound theories to deal with dynamic of land values. In fact, land use is a central concern of classical economics. Ricardo (1821: preface) pointed out: “The produce … is divided among three classes of the commodity, namely, the proprietor of land, the owners of the stock or capital necessary for its cultivation, and laborers by whose industry it is cultivated. But in different stages of the society, the proportions of the whole produce of the earth which will be allotted to each of these classes, under the names of rent, profits, and wages, will be essentially different; depending mainly on the actual fertility of the soil, on the accumulation of capital and population, and on the skill, ingenuity, and the instruments in agriculture." Since the publication of the Principles, many attempts have been done to extend or generalize the system (Barkai, 1959, 1966; Pasinetti, 1960, 1974; Cochrane, 1970; Brems, 1970; Caravale and Tosato, 1980; Casarosa, 1985; Negish, 1989; Morishima, 1989). Nevertheless, what Ricardo (1821: preface) observed long time ago is still relevant today: "To determine the laws which regulate this distribution, is the principal problem in Political Economy: much as the science has been improved by the writings of Turgot, Stuart, Smith, Say, Sismondi, and others, they afford very little satisfactory information respecting the natural course of rent, profit, and wages." In Ricardo's statement there is no reference to land value (price). The traditional Ricardian theory does not determine land price dynamics. Nevertheless, price dynamics are important variables of modern economies. As Cho (1996: 145) stated long time ago, "During the past decade, the number of studies on intertemporal changes in house prices has increased rapidly because of wider availability of extensive micro-level data sets, improvements in modeling techniques, and expanded business applications." The literature on house and land prices has been increasingly expanding since then (e.g., Bryan and Colwell, 1982; Case and Quigley, 1991; Chinloy, 1992; Clapp and Giaccotto, 1994; Calhoun, 1995; Quigley, 1995; Capozza and Seguin, 1996; Alpanda, 2012; Alexander, 2013; Du and Peiser, 2014; Kok et al. 2014). Most of these studies are empirical. There are only a few formal growth models with endogenous land values. According to Liu et al. (2011: 1), "Although it is widely accepted that house prices could have an important influence on macroeconomic fluctuations, quantitative studies in a general equilibrium framework have been scant." Since land value is related to physical wealth which can be accumulated through saving, we need microeconomic mechanism to determine saving behavior. In fact, some studies have been conducted to examine the role of land in economic development and changes of land value. According to Deaton and Laroque (2001): "The user cost of land reduces the resources available for consumption of reproducible goods, so that the introduction of intrinsically valuable land into a growth model lowers the equilibrium stock of capital and raises the equilibrium interest rate. On the asset side, the presence of land causes life-cycle savings to be reallocated away from productive capital towards land. The social optimum in such a model is for land to be nationalized and provided at zero rent. Land markets, far from generating saving and growth, are inimical to capital formation." By including land and its value as endogenous variable the new equilibrium is typically obtained with a lower stock of capital and a higher rate of interest. This is the effect identified by Nichols (1970), Feldstein (1977) and by Drazen and Eckstein (1988), and Deaton and Laroque (2001), and which was first explored by Allais (1948). The effect of depressing the accumulation of productive capital is also pointed in studies by Jappelli and Pagano (1994). In these approaches resources are not introduced into the growth theory with land.

The Ricardian theory does not provide a profound microeconomic mechanism of wealth accumulation. On the other hand, neoclassical growth theory models endogenous wealth accumulation with microeconomic foundation. We will integrate the neoclassical growth theory with the Ricardian theory of distribution for studying dynamic interactions among growth, wealth and income distribution, and economic structures. This paper is concentrated on tradeoffs among economic growth, consumption, resource dynamics, division of labor, and economic structural change. This study attempts to make a contribution to the literature by examining interdependence between savings and dynamics of renewable resources with an alternative approach to consumers' behavior. It is an extension of the growth models proposed by Zhang (2011, 2014). The main difference between this study and Zhang (2011) is that in this paper we make land value as an endogenous variable, while Zhang's 2011 model does not consider this complicated issue. Zhang's 2014 model includes land value as endogenous variable. This paper integrates the unique features of the previous models into a compact framework. This paper is organized as follows. Section 2 introduces the basic model with wealth accumulation and environmental dynamics. Section 3 examines dynamic properties of the model and simulates the model, identifying the existence of a unique equilibrium and checking stability conditions. Section 4 conducts comparative dynamic analysis with regard to changes in the propensity to consume resources, the propensity to consume housing, the propensity to consume agricultural goods, the propensity to consume industrial goods, the propensity to save, the population, and the output elasticity of capital of the resource sector. Section 5 concludes the study. The appendix proves the analytical results in Section 3.

2 The model

The economy has industrial, agricultural, and renewable resource sectors. Most aspects of the production sectors are similar to the standard one-sector growth model in the neoclassical growth theory (Burmeister and Dobell, 1970; Barro and Sala-i-Martin, 1995). Production sectors or firms use physical capital, labor and land inputs. Exchanges take place in perfectly competitive markets. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households. We select commodity to serve as numeraire, with all the other prices being measured relative to its price. We assume that wage rate is identical among all professions. The industrial production is the same as that in Solow's one-sector neoclassical growth model. It is a commodity used both for investment and consumption. The agricultural sector produces agricultural goods, which is used for consumption. The population is homogenous and constant. We neglect effects of changing population on economic structure and land values. The total land is homogenous and constant. The land is owned by households and is distributed between housing and agricultural production in free land market. The assumption of fixed land is a strict requirement. As observed by Glaeser, et al. (2005), land supply elasticity varies substantially over space in the USA (see also, Davis and Heathcote, 2007). This study neglects possible changes in land supply. Households achieve the same utility level regardless of what profession they choose. All the markets are perfectly competitive. We select industrial goods to serve as numeraire.

The industrial sector

We assume that production is to combine labor force, $N_i(t)$, and physical capital, $K_i(t)$. We use the conventional production function to describe a relationship between inputs and output. The production function $F_i(t)$ is specified as follows

$$F_i(t) = A_i K_i^{\alpha_i}(t) N_i^{\beta_i}(t), \;\; A_i, \alpha_i, \beta_i > 0, \;\; \alpha_i + \beta_i = 1, \tag{1}$$

where A_i, α_i and β_i are parameters. The production function is a neoclassical one and homogeneous of degree one with the inputs. Markets are competitive; thus labor and capital earn their marginal products. The rate of interest $r(t)$ and wage rate $w(t)$ are determined by markets. The marginal conditions are given by

$$r(t) + \delta_k = \frac{\alpha_i F_i(t)}{K_i(t)}, \;\; w(t) = \frac{\beta_i F_i(t)}{N_i(t)}, \tag{2}$$

where δ_k is the fixed depreciation rate of physical capital.

The agricultural sector

We assume that agricultural production is carried out by combination of capital $K_a(t)$, labor force $N_a(t)$, and land $L_a(t)$ as follows

$$F_a(t) = A_a K_a^{\alpha_a}(t) N_a^{\beta_a}(t) L_a^{\varsigma}(t), \;\; A_a, \alpha_a, \beta_a, \varsigma > 0, \;\; \alpha_a + \beta_a + \varsigma = 1, \tag{3}$$

where $L_a(t)$ is the land employed by the agricultural sector, and A_a, α_a, β_a, and ς are parameters. The marginal conditions are given by

$$r(t) + \delta_k = \frac{\alpha_a p_a(t) F_a(t)}{K_a(t)}, \;\; w(t) = \frac{\beta_a p_a(t) F_a(t)}{N_a(t)}, \;\; R(t) = \frac{\varsigma p_a(t) F_a(t)}{L_a(t)}, \tag{4}$$

where $p_a(t)$ is the price of agricultural goods and $R(t)$ is the land rent.

Choice between physical wealth and land

We now model dynamics of land value on the basis of Zhang (2014). Land may be owned by different agents under various institutions. This study considers the case that land is privately owned by households. There are different approaches with regard to determination of land prices and rents. For instance, some studies (Iacoviello, 2005; Iacoviello and Neri, 2010) assume that households are credit constrained and these households use land or houses as collateral to finance consumption expenditures. These models with credit-constrained households are used to explain positive co-movements between house prices and consumption expenditures (see also, Campbell and Mankiw, 1989; Zeldes, 1989; Case, *et al.*, 2005; Mian and Sufi, 2010; Oikarinen, 2014). In Liu *et al.* (2011), instead of households, firms are assumed to be credit constrained. Firms finance investment spending by using land as a collateral asset. Land can be sold and bought in free markets without any friction and transaction costs. Land use will not waste land and land cannot regenerate itself. Households own land and physical wealth. We use $p_L(t)$ to denote the price of land. Consider now an investor with one unity of money. He can either invest in capital good thereby earning a profit equal to the net own-rate of return $r(t)$ or invest in land thereby earning a profit equal to the net own-rate of return $R(t)/p_L(t)$. As we assume capital and land markets to be at competitive equilibrium at any point in time, two options must yield equal returns, i.e.

$$\frac{R(t)}{p_L(t)} = r(t). \tag{5}$$

This equation enables us to determine choice between owning land and wealth. This assumption is made under many strict conditions. For instance, we neglect any transaction costs and any time needed for buying and selling. Expectations on land are complicated. Equation (5) also implies perfect information and rational expectation. It should be noted we don't follow traditional approaches to determination of land value. According to Goodwin *et al*. (2003: 744) "most existing models of agricultural asset values adopt an income approach whereby the value of an asset is modeled as the present value of expected future cash flows, discounted appropriately to reflect the risk of each source of earnings. We will argue that problems occur when this model is used in empirical applications aimed at investigating the factors that determine the value of farmland."

Change of renewable resources

We model resources dynamics on the basis of Zhang (2011). Let $X(t)$ stand for the stock of the resource. We are concerned with a single kind of resource. It is well known that the logistic model has been frequently used in the literature of growth with renewable resource (e.g., Brander and Taylor, 1997; Brown, 2000; Hannesson, 2000; Cairns and Tian, 2010; and Farmer and Bednar-Friedl, 2011). The natural growth rate of the resource is taken on the following logistic function

$$\phi_0 \, X(t)\left(1 - \frac{X(t)}{\phi(L_x(t))}\right),$$

where the variable, $\phi(L_x(t))$, is the maximum possible size for the resource stock, called the carrying capacity of the resource, and the variable, ϕ_0, is "uncongested" or "intrinsic" growth rate of the renewable resource. If the stock is equal to ϕ, then the growth rate should equal zero. If the carrying capacity is much larger than the current stock, then the growth rate per unit of the stock is approximately equal to the intrinsic growth rate. That is, the congestion effect is negligible. In this study, for simplicity we assume the intrinsic growth rate constant. We require $d\phi / dL_x \geq 0$. If the resource is forest, it is obvious that more land implies high capacity. It should be noted that there are some alternative approaches to renewable resources. For instance, Tornell and Velasco (1992), Long and Wang (2009), and Fujiwara (2011) use linear resource dynamics. In the literature of resource economics, different factors are introduced to make the capacity as an endogenous variable. For instance, Benchekroun (2003) assumes an inversed-V shaped dynamics of resource accumulation, namely, the resource decreases if its stock is sufficiently large. We may consider the capacity dependent on some factors such as efforts. For instance, in the case of forestry fertilizers or cleaning activities of the soil may affect the parameter. With aquaculture, we can also refer to feedings schemes, water temperature, or oxygen levels. See Levhari and Withagen (1992) for how to introduce human efforts to the dynamics of resources. Let $F_x(t)$ stand for the harvest rate of the resource. The change rate in the stock is then equal to the natural growth rate minus the harvest rate, that is

$$\dot{X}(t) = \phi_0 \, X(t)\left(1 - \frac{X(t)}{\phi(L_x)}\right) - F_x(t). \qquad (6)$$

We now examine functional form of the harvest rate. We assume a nationally owned open-access renewable resource (which was initially examined by Gordon, 1954). Recent approaches to growth with renewable resources with different property-rights regimes are referred to, for instance, Alvarez-Guadrado and VonLong (2011). With open access, harvesting occurs up to the point at which the current return to a representative entrant equals the entrant's cost. Aside from the stock of the renewable resources, like the good sector there are two factors of production. We use $N_x(t)$ and $K_x(t)$ to stand for the labor force and capital stocks employed by the resource sector. We assume that harvesting of the resource is carried out according to the following harvesting production function

$$F_x(t) = A_x X^b(t) L_x^{b_x}(t) K_x^{\alpha_x}(t) N_x^{\beta_x}(t), \quad A_x, x, b, b_x, \alpha_x, \beta_x > 0, \quad \alpha_x + \beta_x = 1, \quad (7)$$

where A_x, m_x, b, b_x, α_x and β_x are parameters. The specified form implies that if the capital (like machine) and labor inputs are simultaneously doubled, then harvest is also doubled for given levels of technology and resource at any time. It should be noted that the following Schaefer harvesting production function (Schaefer, 1957)

$$F_x(t) = A_x X(t) N_x(t),$$

is evidently a special case of (6). The Schaefer production function does not take account of capital (or with capital being fixed) and technology. The function with fixed capital and technology is widely applied to fishing (see also, Paterson and Wilen, 1977; Milner-Gulland and Leader-Williams, 1992; Bulter and van Kooten, 1999). As machines are important inputs in harvesting, we explicitly take account of capital input. We use $p_x(t)$ to denote the price of the resource. The marginal conditions are given as follows

$$r(t) + \delta_k = \frac{\alpha_x p_x(t) F_x(t)}{K_x(t)}, \quad w(t) = \frac{\beta_x p_x(t) F_x(t)}{N_x(t)}. \quad (8)$$

Consumer behavior

For simplicity, we use lot size to stand for housing. As argued, for instance, by Davis and Heathcote (2007), most of the fluctuations in house prices are driven by land price rather than by the cost of structures. Consumers decide consumption levels of industrial and agricultural goods and lot size, as well as on how much to save. This study uses the approach to consumers' behavior proposed by Zhang (1993). We denote respectively physical wealth by $\bar{k}(t)$ and land $\bar{l}(t)$ owned by the representative household. The total value of wealth owned by the household $a(t)$ is the sum of the two assets

$$a(t) = \bar{k}(t) + p_L(t)\bar{l}(t). \quad (9)$$

Per capita current income from the interest payment $r(t)\bar{k}(t)$, the wage payment $w(t)$, and the land revenue $R(t)\bar{l}(t)$ is given by

$$y(t) = r(t)\bar{k}(t) + w(t) + R(t)\bar{l}(t). \qquad (10)$$

We call $y(t)$ the current income. The per capita disposable income is given by

$$\hat{y}(t) = y(t) + a(t). \qquad (11)$$

The disposable income is used for saving and consumption. It should be remarked that in the growth literature, for instance, in the Solow model, the saving is out of the current income, $y(t)$, while in this study the saving is out of the disposable income which is dependent both on the current income and the value of wealth. The implications of our approach are similar to those in the Keynesian consumption function and models based on the permanent income hypothesis, which are empirically much more valid than the approaches in the Solow model or the in Ramsey model. The approach to household behavior in this study is discussed at length by Zhang (2005).

At each point of time, a consumer would distribute the total available budget among saving, $s(t)$, consumption of the commodity, $c_i(t)$, consumption of the resource good, $c_x(t)$, consumption of the agricultural good, $c_a(t)$, and housing, $l_h(t)$. The budget constraint is given by

$$c_i(t) + s(t) + p_a(t)c_a(t) + p_x(t)c_x(t) + R(t)l_h(t) = \hat{y}(t). \qquad (12)$$

In our model, at each point of time, consumers have five variables, $s(t)$, $c_i(t)$, $c_x(t)$, $c_a(t)$, and $l_h(t)$, to decide. The consumer's utility function is specified as follows

$$U(t) = c_i^{\xi_0}(t)c_a^{\mu_0}(t)c_x^{\chi_0}(t)l_h^{\eta_0}(t)s^{\lambda_0}(t), \ \ \xi_0, \mu_0, \chi_0, \eta_0, \lambda_0 > 0,$$

in which $\xi_0, \mu_0, \chi_0, \eta_0$, and λ_0 are the urban household's elasticity of utility with regard to the commodity, the agricultural goods, the resource, housing, and saving. We call $\xi_0, \mu_0, \chi_0, \eta_0$, and λ_0 propensities to consume the commodity, the agricultural goods, the resource, and housing, and to hold wealth, respectively. Maximizing $U(t)$ subject to the budget constraint (12) yields

$$c_i(t)=\xi\,\hat{y}(t),\;\; p_a(t)c_a(t)=\mu\,\hat{y}(t),\;\; p_x(t)c_x(t)=\chi\,\hat{y}(t),\;\; R(t)l_h(t)=\eta\,\hat{y}(t),\;\; s(t)=\lambda\,\hat{y}(t), \qquad (13)$$

where

$$\xi\equiv\rho\xi_0,\;\; \mu\equiv\rho\mu_0,\;\; \chi\equiv\rho\chi_0,\;\; \eta\equiv\rho\eta_0,\;\; \lambda\equiv\rho\lambda_0,\;\; \rho\equiv\frac{1}{\xi_0+\mu_0+\chi_0+\eta_0+\lambda_0}.$$

The demand for the resource good is given by $c_x(t)=\chi\,\hat{y}(t)/\,p_x(t)$. The demand decreases in its price and increases in the disposable income. An increase in the propensity to consume the resource good increases the consumption when the other conditions are fixed.

Wealth accumulation

According to the definition of $s(t)$, the change in the household's wealth is given by

$$\dot{a}(t)=s(t)-a(t). \qquad (14)$$

The equation simply states that the change in wealth is equal to saving minus dissaving.

Balances of demand and supply for industrial goods

Demand and supply for the industrial sector's output balance at any point of time

$$c(t)N+s(t)N+\delta_k K(t)=F_i(t)+K(t). \qquad (15)$$

Balances of demand and supply for agricultural goods

The demand and supply for the agricultural sector's output balance at any point in time

$$C_a(t)=c_a(t)N=F_a(t). \qquad (16)$$

Balances of demand and supply for renewable resources

The demand and supply for the resource balance at any point of time

$$c_x(t)N=F_x(t). \qquad (17)$$

All the land owned by households

The land owned by the population is equal to the national available land

$$\bar{l}(t)N = L. \tag{18}$$

Full employment of capital

We use $K(t)$ to stand for the total capital stock. We assume that the capital stock is fully employed. We have

$$K_i(t) + K_a(t) + K_x(t) = K(t). \tag{19}$$

The value of physical wealth and capital

The value of physical capital is equal to the value of physical wealth

$$\bar{k}(t)N = K(t). \tag{20}$$

Full employment of labor force

We assume that labor force is fully employed

$$N_i(t) + N_a(t) + N_x(t) = N. \tag{21}$$

The land market clearing condition

The land is fully used

$$l_h(t)N + L_a(t) + L_x(t) = L. \tag{22}$$

Land use for renewable resources

The land use for residents and agricultural product are determined respectively by the marginal conditions for the household and the agricultural sector. We now introduce a mechanism to decide the amount of land used for renewable resource. The land of renewable resource is assumed to be

$$L_x(t) = \varphi L_a(t). \tag{23}$$

where φ is a constant parameter. This assumption is accepted mainly for convenience of analysis.

We thus built the model. The model describes dynamics of the economic structure and values of lan and renewable resources. We now examine dynamic properties of the model.

3 The dynamics and the motion by simulation

The economic system contains many nonlinear relations. It is difficult to analytically explore the properties of the nonlinear dynamic system. For illustration, we simulate the model. The appendix shows that the dynamics of the national economy can be expressed as two differential equations. First, we introduce a variable $z(t)$ by

$$z(t) \equiv \frac{r(t) + \delta_k}{w(t)}.$$

We now show that the dynamics can be expressed by two differential equations with $z(t)$ and $X(t)$ as the variables.

Lemma

The motion of the system is determined by the following two differential equations

$$\dot{z}(t) = \Lambda(z(t)),$$
$$\dot{X}(t) = \Omega(z(t),\ X(t)), \qquad (24)$$

where the right-hand sides of (24) are functions of $z(t)$ and $X(t)$ determined in the appendix. Moreover, all the other variables are determined as functions of $z(t)$ and $X(t)$ at any point in time by the following procedure: $r(t)$ and $w(t)$ by (A2) $\rightarrow$ $\bar{k}(t)$ by (A24) $\rightarrow$ $K_a(t)$ by (A18) $\rightarrow$ $K_i(t)$ and $K_x(t)$ by (A21) $\rightarrow$ $N_i(t)$, $N_x(t)$ and $N_a(t)$ by (A1) $\rightarrow$ $\hat{y}(t)$ by (A14) $\rightarrow$ $\bar{l}$ by (18) $\rightarrow$ $R(t)$ by (A13) $\rightarrow$ $p_L(t)$ by (A23) $\rightarrow$ $a(t)$ by (A25) $\rightarrow$ L_a, L_x and l_h by (A11) $\rightarrow$ $p_a(t)$ by (A5) $\rightarrow$ $F_i(t)$ by (1) $\rightarrow$ $F_a(t)$ by (3) $\rightarrow$ $F_x(t)$ by (7) $\rightarrow$ $p_x(t)$ by (8) $\rightarrow$ $c_i(t)$, $c_a(t)$, $c_x(t)$, and $s(t)$ by (13).

The lemma shows that once we determine the values of $z(t)$ and $X(t)$, we can determine all the variables in the economic system. The lemma is important as it gives a procedure to follow the motion of the system with computer. As the expressions of the analytical results are tedious, for illustration we specify the parameter values and simulate the model. We specify the parameters as follows

$$N = 5,\ L = 10,\ \alpha_i = 0.3,\ \alpha_a = 0.1,\ \beta_a = 0.2,\ \alpha_x = 0.34,\ A_i = 1,\ A_a = 0.5,\ A_x = 0.5,$$
$$\lambda_0 = 0.5,\ \xi_0 = 0.07,\ \chi_0 = 0.02,\ \eta_0 = 0.01,\ \mu_0 = 0.02,\ \varphi = 1.6,\ \phi = 4,\ \phi_0 = 5,$$
$$b = 0.7,\ b_x = 0.01,\ \delta_k = 0.05. \quad (25)$$

The population is fixed at 5 and the land is 10. We assume that the propensity to save is much higher than the propensity to consume industrial goods, resources, and agricultural goods. As shown in the appendix, the following variables are invariant in time

$$l_h = 0.43,\ L_a = 3.02,\ L_x = 4.83,\ \bar{l} = 2.$$

We specify the initial conditions as follows

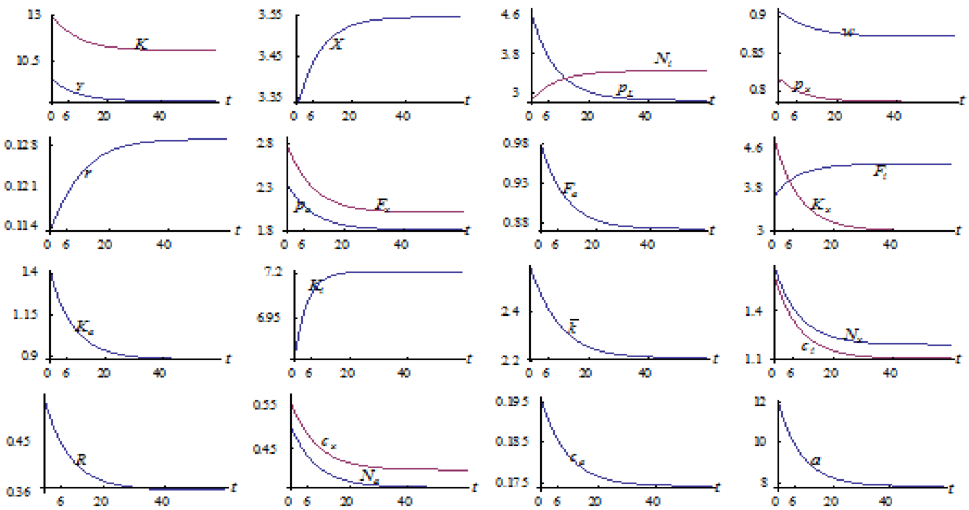
$$X(0) = 3.4,\ z(0) = 0.18.$$

We plot the motion of the variables in Figure 1. In Figure 1 the national gross product (GDP) is

$$Y(t) = F_i(t) + p_a(t)F_a(t) + p_x(t)F_x(t) + l_h\, N\, R(t).$$

The GDP and national capital stock fall over time till they become stationary. The stock of resources rises. The wage rate, price of land, price of resource, price of agricultural goods, and land rent are reduced, and the rate of interest is enhanced. The output level of the agricultural sector is increased and the output level of the industrial sector is reduced. Some of the force is shifted from the industrial sector to the agricultural sector. The capital inputs of the two sectors are increased. The physical wealth, total wealth, and consumption levels of the two goods are increased. It should be noted that the dynamic relationship between the GDP and the land price plotted in Figure 1 is similar to the phenomenon described by Liu *et al*. (2011: 1): “The recent financial crisis caused by a collapse of the housing market propelled the U.S. economy into the Great Recession. A notable development during the crisis period was a slump in business investment in tandem with a sharp decline in land prices.” The conclusions made by Liu *et al*. are based on the data for the Great Recession period as well as for the entire sample period from 1975 to 2010. Our comparative dynamic analysis in the rest of the paper also shows similar conclusions.

Figure 1 The Motion of the Economic System



From Figure 1 we observe that all the variables tend to become stationary in the long term. This implies the existence of some equilibrium point. We confirm the existence of a unique equilibrium point as follows

$$Y = 8.24,\ \ K = 11.07,\ \ X = 3.55,\ \ w = 0.87,\ \ p_L = 2.83,\ \ R = 0.37,\ \ r = 0.13,\ \ p_a = 1.81,$$

$$p_x = 0.79,\ \ F_a = 0.87,\ \ F_i = 4.3,\ \ F_x = 2.01,\ \ K_a = 0.88,\ \ K_i = 7.21,\ \ K_x = 2.99,$$

$$N_a = 0.36,\ \ N_i = 3.45,\ \ N_x = 1.19,\ \ L_a = 3.02,\ \ L_x = 4.83,\ \ \bar{l} = 2,\ \ l_h = 0.43,\ \ \bar{k} = 2.22,$$
$$c_a = 0.17,\ \ c_x = 0.4,\ \ c_i = 1.1,\ \ a = 7.88. \qquad (26)$$

The eigenvalues at the equilibrium point are

$$-4.265,\ \ -0.116.$$

This guarantees the stability of the steady state. This result is important as it guarantees the relevance of comparative dynamic analysis in the next section.

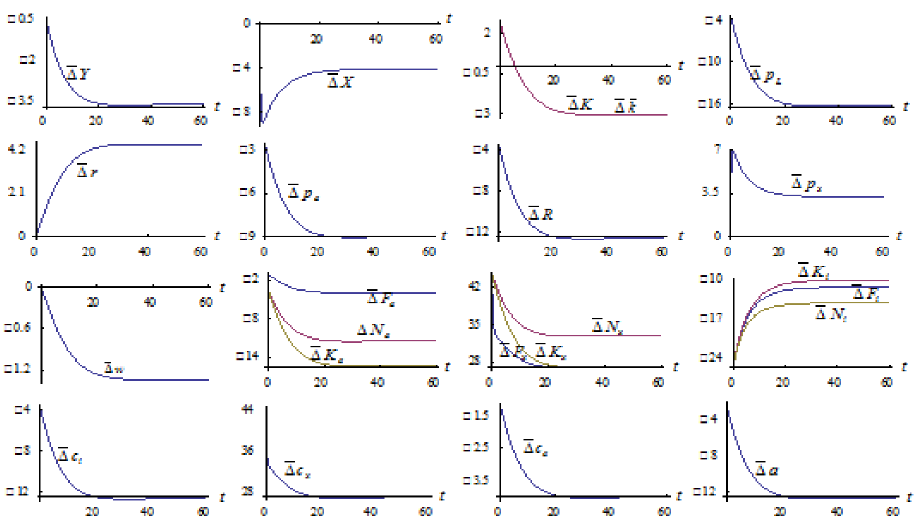
4 Comparative dynamic analysis

We now examine effects of changes in some parameters on the motion of the economic system. As the lemma gives a computational procedure to calibrate the motion of all the variables and the equilibrium point is locally stable, it is straightforward to conduct comparative dynamic analysis. In the rest of this study we use $\bar{\Delta} x_j(t)$ to stand for the change rate of the variable, $x_j(t)$, in percentage due to changes in a parameter value.

A rise in the propensity to consume resources

We first examine the effects of the following change in the propensity to consume resources: $\chi_0 : 0.02 \Rightarrow 0.03$. The land use pattern is not affected by the change in the preference. The effects on the other variables are plotted in Figure 2. The household consumes more resources and the stock of resources is reduced. The price of resources is enhanced. The household reduces the consumption levels of agricultural and industrial goods. The household's physical wealth and national physical capital rise initially and fall in the long term. The GDP is reduced. The land rent and land value are lowered. The price of agricultural goods is reduced. The wage rate falls in tandem with rising in the rate of interest. The output level and two inputs of the industrial and agricultural sectors are lowered. The output level and two inputs of the resource sector are augmented. Debates about whether natural resources are a blessing or a curse for human development are still a hot topic in the literature of economic development.

Figure 2 A Rise in the Propensity to Consume Resources



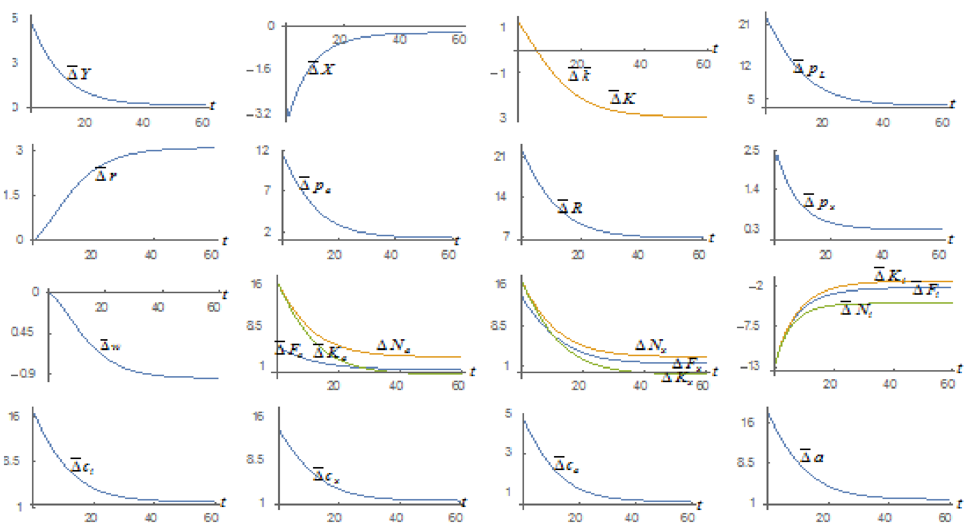
A rise in the propensity to consume housing

The propensity to consume housing is shifted as follows: $\eta_0 : 0.01 \Rightarrow 0.015$. The land use is re-distribution as follow

$$\overline{\Delta} L_a = \overline{\Delta} L_x = -1.07, \ \ \overline{\Delta} l_h = 3.88, \ \ \overline{\Delta} \bar{l} = 0.$$

The household has larger house size. The resource and agricultural sectors use less land. The effects on the other variables are plotted in Figure 3. The land rent and land value are augmented. The price of agricultural goods and price of resources are increased. The wage rate is lowered in tandem with rising in the rate of interest. The household's physical wealth and national total capital stocks are augmented initially and reduced in the long term. The household consumes more agricultural goods, resources, and goods. The household also owns more wealth. The national output rises. The output level and two inputs of the industrial sector are lowered. The output level and two inputs of the resource and agricultural sectors are augmented.

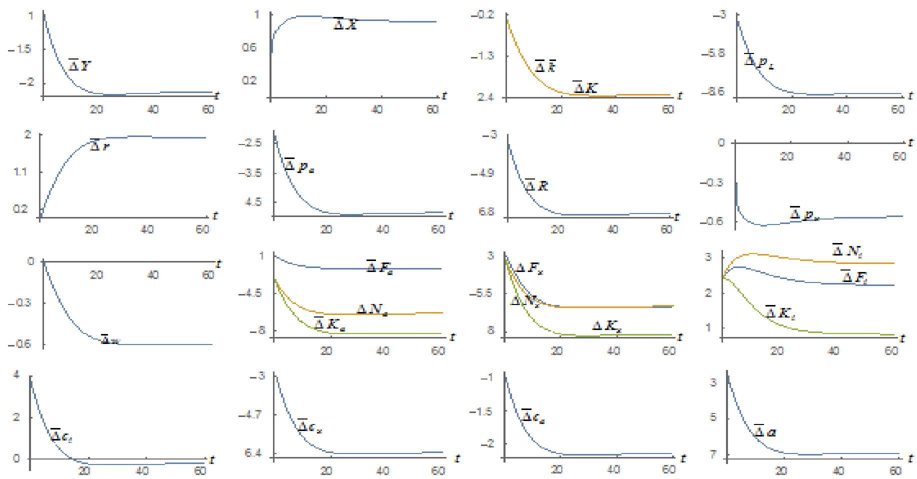
Figure 3 A Rise in the Propensity to Consume Housing



The propensity to consume industrial goods being enhanced

We now study the effects that the following change in the propensity to consume industrial goods: $\xi_0 : 0.07 \Rightarrow 0.075$. The land use pattern is not affected. The effects on the other variables are plotted in Figure 4. As the household spends more out of the disposable income on consuming industrial goods, the total capital stock and the GDP are lowered. The household owns less physical wealth and wealth. The households consume more industrial goods and less resources and agricultural goods. The stock of resource is increased in association with falling price of the resource. The wage rate is reduced and the rate of interest is enhanced. Both the land value and the land rent are reduced. The price of agricultural goods falls. The output levels and two inputs of the agricultural and resource sectors reduced. In the long term the increase in the output of the industrial sector is due to the reallocation of labor force from the agricultural sector to the industrial sector. The capital inputs of the two sectors are reduced in the long term.

Figure 4 A Rise in the Propensity to Consume Industrial Goods



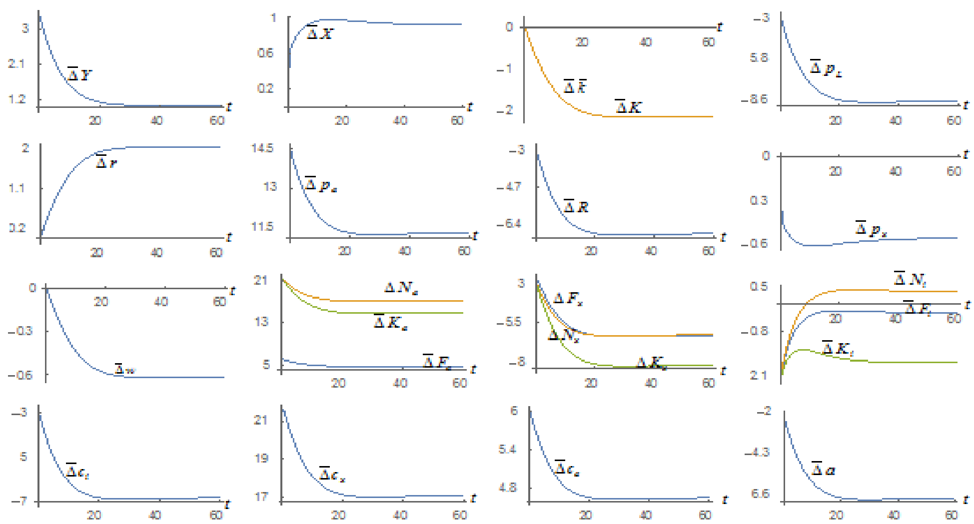
A rise in the propensity to consume agricultural goods

We now study the effects that the propensity to consume agricultural goods is increased as follows: $\mu_0 : 0.02 \Rightarrow 0.025$. The impact on land use pattern is as follows

$$\overline{\Delta} L_a = \overline{\Delta} L_x = 4.51, \ \ \overline{\Delta} l_h = -16.4, \ \ \overline{\Delta} \bar{l} = 0.$$

More land is devoted to agricultural and resource supplies and less to housing. The effects on the system over time are plotted in Figure 5. As the household spends more out of the disposable income on consuming agricultural goods, the total capital stock is reduced. The GDP is augmented. The wage rate is reduced and the rate of interest is enhanced. The land rent and the value of land are lowered. The household holds less wealth and physical wealth. The household's consumption level of agricultural goods rises. The household's consumption level of industrial goods falls. The price of agricultural goods rises. The output level and two input factors of the agricultural sector are augmented. The output level and capital input of the industrial sector are reduced. The stock of resources is increased in association with falling in the resource price. The output level and the two inputs of the resource sector are reduced.

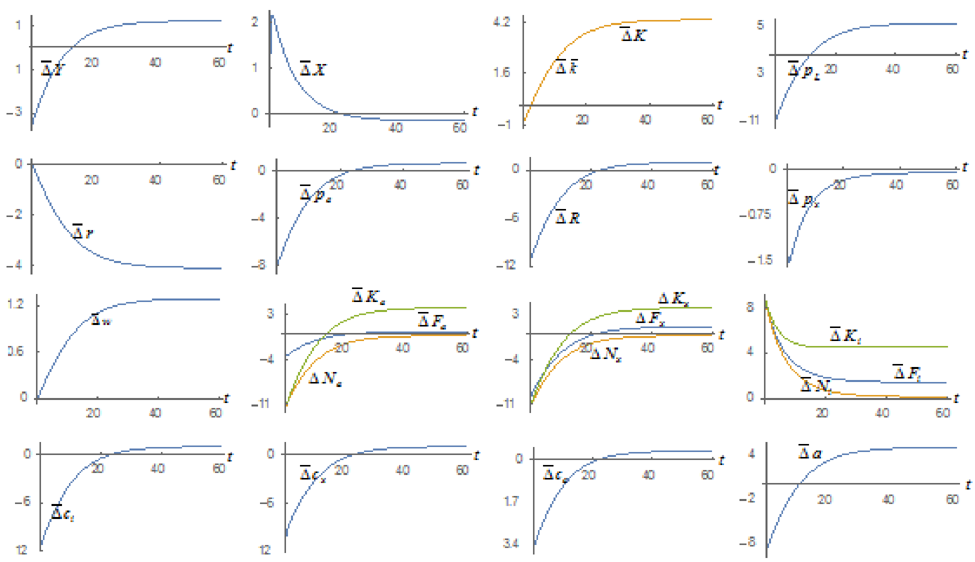
Figure 5 The Propensity to Consume Agricultural Goods Being Enhanced



A rise in the propensity to save

We now change the propensity to save as follows: $\lambda_0 : 0.5 \Rightarrow 0.52$. The land use pattern is not affected. The effects on the variables over time are plotted in Figure 6. As the household tends to save more out of the disposable income, the physical wealth falls initially and rise in the long term. The GDP falls initially and rises in the long term. The household consumes industrial goods, agricultural goods, and resource less initially and more in the long term. The household owns wealth less initially and more in the long term. The land value falls initially and rises in the long term. The land rent and the price of agricultural goods are reduced initially and increased in the long term. The wage rate rises and the rate of interest falls. The output and two input factors of the industrial sectors are expanded. The output and two capital input of the agricultural sectors are reduced initially and expanded in the long term. The stock of resources is augmented initially and reduced in the long term. The price of resources falls.

Figure 6 The Propensity to Save Being Enhanced



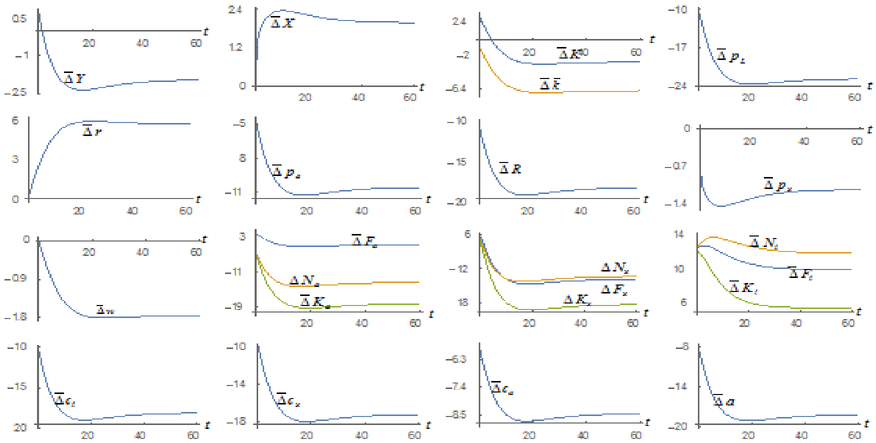
The population being augmented

We now study the effects that the population is expanded as follows: $N: 5 \Rightarrow 5.2$. The impact on land use pattern is as follows

$$\bar{\Delta} L_a = \bar{\Delta} L_x = 0, \ \ \bar{\Delta} l_h = \bar{\Delta} \bar{l} = -3.85.$$

The land inputs of the agricultural and resource sectors are not affected. The lot size is increased. The household owns less land. The effects on the system over time are plotted in Figure 7. As the nation has more people, the total capital stock and GDP are increased initially and reduced in the long term. The wage rate is reduced and the rate of interest is enhanced. The land rent and the value of land are lowered. The household holds less wealth and physical wealth. The household's consumption levels of agricultural goods, industrial goods and resource all fall. The price of agricultural goods is lowered. The output levels and two inputs of the agricultural and resource sectors are reduced. The output level and two inputs of the industrial sector are augmented. The stock of resources is increased in association with falling in the resource price.

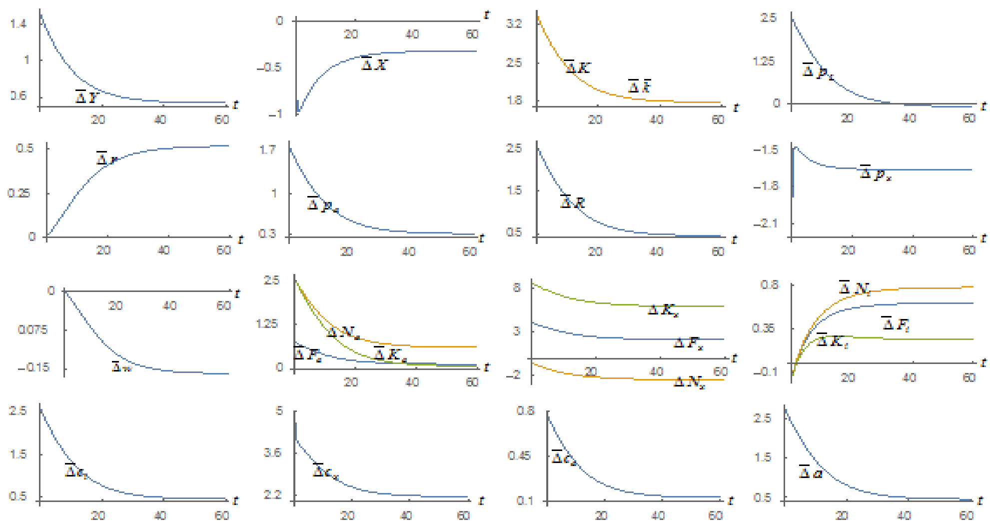
Figure 7 The Population Being Augmented



A rise in the output elasticity of capital of the resource sector

We now examine effects of the following rise in the output elasticity of capital of the resource sector: $\alpha_x : 0.34 \Rightarrow 0.36$. The land use pattern is not affected. The effects on the other variables are plotted in Figure 8. A rise in this parameter implies that the capital share of the total factor cost is increased in the optimal decision. The total capital and the GDP are increased. The rate of interest, the price of agricultural goods, the land rent, and land value are all increased. The wage rate and price of resources are reduced.

Figure 8 A Rise in the Output Elasticity of Capital of the Resource Sector



5. Concluding remarks

This study examined dynamic interactions among land, renewable resource, capital and economic structure. The main framework is neoclassical and the household's decision is based on an alternative approach proposed by Zhang. We integrated some ideas in the neoclassical growth theory, resource economics and land economics in a compact framework. By simulation, we demonstrated that the economic system has a unique steady state. We also conducted comparative dynamic analysis with regard to changes in the propensity to consume resources, the propensity to consume housing, the propensity to consume agricultural goods, the propensity to consume industrial goods, the propensity to save, the population, and the output elasticity of capital of the resource sector. Our results on relations between economic growth and land price provide some insights into the phenomenon that is described by Liu *et al*. (2011: 1), "The recent financial crisis caused by a collapse of the housing market propelled the U.S. economy into the Great Recession. A notable development during the crisis period was a slump in business investment in tandem with a sharp decline in land prices." Our model is built under many strict conditions without taking account of many possible important determinants of land and resource prices. These limitations become apparent in the light of the sophistication of the literature of growth theory, resource and land economics. This implies the necessity of extending or generalizing the model. For instance, we may generalize the model by using more general function forms of the three sectors and the utility function. It is also possible to extend the model by taking account of heterogeneity of households.

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Appendix: Proving the Lemma

The appendix shows that the dynamics can be expressed by two differential equations. From (2), (4) and (8), we obtain

$$z \equiv \frac{r+\delta_k}{w} = \frac{\tilde{\alpha}_i N_i}{K_i} = \frac{\tilde{\alpha}_a N_a}{K_a} = \frac{\tilde{\alpha}_x N_x}{K_x}, \tag{A1}$$

where we omit time index and $\tilde{\alpha}_j \equiv \alpha_j / \beta_j$, $j = i, a, x$. By (1) and (2), we have

$$r+\delta_k = \frac{\alpha_i A_i z^{\beta_i}}{\tilde{\alpha}_i^{\beta_i}}, \quad w = \frac{\tilde{\alpha}_i^{\alpha_i} \beta_i A_i}{z^{\alpha_i}}, \tag{A2}$$

where we also use (A1). We express w and r as functions of z.

From (13) and (16), we get

$$\mu \hat{y} N = p_a F_a . \tag{A3}$$

From (4), we have

$$r+\delta_k = \frac{\alpha_a p_a F_a}{K_a}. \tag{A4}$$

From (A4) and (3) we solve

$$p_a \left(\frac{L_a}{K_a}\right)^{\varsigma} = \frac{\tilde{\alpha}_a^{\beta_a}(r+\delta_k)}{\alpha_a A_a z^{\beta_a}}, \tag{A5}$$

where we use (A1). From (A3) and (A4), we solve

$$\mu \hat{y} N = \left(\frac{r+\delta_k}{\bar{\tau}_a \alpha_a}\right) K_a . \tag{A6}$$

By (4) and (A3), we have

$$R = \frac{\varsigma \mu \hat{y} N}{L_a}. \tag{A7}$$

From $R l_h = \eta \hat{y}$ in (13) and (A7), we have

$$\varsigma \mu N l_h = \eta L_a . \tag{A8}$$

Insert (23) in (22)

$$l_h N + (1+\varphi) L_a = L . \tag{A9}$$

From (A8), (A9) and (23) we solve the land distribution as follows

$$l_h = \frac{\eta L}{\eta N + (1+\varphi)\varsigma \mu N}, \quad L_a = \frac{\varsigma \mu N l_h}{\eta}, \quad L_x = \varphi L_a . \tag{A10}$$

The land distribution is invariant over time.

From the definition of $\hat{y}$, we have

$$\hat{y} = (1+r)\bar{k} + w + R\bar{l} + p_L \bar{l} . \tag{A11}$$

Insert (5) in (A11)

$$\hat{y} = (1+r)\bar{k} + w + \left(1+\frac{1}{r}\right)\bar{l} R, \tag{A12}$$

From $R l_h = \eta \hat{y}$ in (13) and (A12) we solve

$$R = \omega_1 \bar{k} + \omega_2 , \tag{A13}$$

where

$$\omega_1(z) \equiv (1+r)\left[\frac{l_h}{\eta} - \left(1+\frac{1}{r}\right)\bar{l}\right]^{-1}, \quad \omega_2(z) \equiv w\left[\frac{l_h}{\eta} - \left(1+\frac{1}{r}\right)\bar{l}\right]^{-1}.$$

From (A12) and (A13) we have

$$\hat{y} = \tilde{\omega}_1 \bar{k} + \tilde{\omega}_2 , \tag{A14}$$

where

$$\tilde{\omega}_1(z) \equiv (1+r) + \left(1+\frac{1}{r}\right)\bar{l}\,\omega_1, \quad \tilde{\omega}_2(z) \equiv w + \left(1+\frac{1}{r}\right)\bar{l}\,\omega_2 .$$

Insert (A1) in $N_i + N_a + N_x = N$

$$\frac{K_i}{\tilde{\alpha}_i} + \frac{K_a}{\tilde{\alpha}_a} + \frac{K_x}{\tilde{\alpha}_x} = \frac{N}{z}. \tag{A15}$$

From (19) and (20) we have

$$K_i + K_a + K_x = N\bar{k}. \tag{A16}$$

Insert (4) in (A3)

$$\mu \hat{y} N = \left(\frac{r+\delta_k}{\alpha_a}\right) K_a. \tag{A17}$$

From (A17) and (A14) we solve

$$K_a = \hat{\omega}_1 \bar{k} + \hat{\omega}_2, \tag{A18}$$

where

$$\hat{\omega}_1(z) \equiv \tilde{\omega}_1 \mu N \left(\frac{\alpha_a}{r+\delta_k}\right), \quad \hat{\omega}_2(z) \equiv \tilde{\omega}_2 \mu N \left(\frac{\alpha_a}{r+\delta_k}\right).$$

Insert (A18) in, respectively, (A15) and (A16)

$$\frac{K_i}{\tilde{\alpha}_i} + \frac{K_x}{\tilde{\alpha}_x} = b_1 \equiv \frac{N}{z} - \frac{\hat{\omega}_2}{\tilde{\alpha}_a} - \frac{\hat{\omega}_1 \bar{k}}{\tilde{\alpha}_a},$$
$$K_i + K_x = b_2 \equiv N\bar{k} - \hat{\omega}_1 \bar{k} - \hat{\omega}_2. \tag{A19}$$

Solve (A19)

$$K_i = \alpha_0 b_1 - \frac{\alpha_0 b_2}{\tilde{\alpha}_x}, \quad K_x = \frac{\alpha_0 b_2}{\tilde{\alpha}_i} - \alpha_0 b_1, \tag{A20}$$

where

$$\alpha_0 \equiv \left(\frac{1}{\tilde{\alpha}_i} - \frac{1}{\tilde{\alpha}_x}\right)^{-1}.$$

Insert the definitions of b_j in (A20)

$$K_i = m_i \bar{k} - \bar{m}_i, \quad K_x = m_x \bar{k} - \bar{m}_x, \tag{A21}$$

where

$$m_i(z) \equiv -\frac{\alpha_0\,\hat{\omega}_1}{\tilde{\alpha}_a} - \frac{\alpha_0}{\tilde{\alpha}_x} N + \frac{\alpha_0}{\tilde{\alpha}_x}\hat{\omega}_1, \quad \overline{m}_i(z) \equiv \frac{\alpha_0\,\hat{\omega}_2}{\tilde{\alpha}_a} - \frac{\alpha_0\,N}{z} - \frac{\alpha_0}{\tilde{\alpha}_x}\hat{\omega}_2,$$

$$m_x(z) \equiv \frac{\alpha_0\,N}{\tilde{\alpha}_i} - \frac{\alpha_0\,\hat{\omega}_1}{\tilde{\alpha}_i} + \frac{\alpha_0\,\hat{\omega}_1}{\tilde{\alpha}_a}, \quad \overline{m}_x(z) \equiv \frac{\alpha_0\,\hat{\omega}_2}{\tilde{\alpha}_i} + \frac{\alpha_0\,N}{z} - \frac{\alpha_0\,\hat{\omega}_2}{\tilde{\alpha}_a}.$$

By (A18) and (A21), we solve the capital distribution as functions of z and $\bar{k}$. By (A1), we solve the labor distribution as functions of z and $\bar{k}$ as follows

$$N_i = \frac{z\,K_i}{\tilde{\alpha}_i}, \quad N_a = \frac{z\,K_a}{\tilde{\alpha}_a}, \quad N_x = \frac{z\,K_x}{\tilde{\alpha}_x}. \tag{A22}$$

From (5)

$$p_L = \frac{R}{r}. \tag{A23}$$

Insert (13) in (17)

$$\chi\,\hat{y}\,N_0 = \left(\frac{r+\delta_k}{\alpha_x}\right) K_x, \tag{A24}$$

where we also (8). Insert (A14) and (A21) into (A24)

$$\bar{k} = \left(\tilde{\omega}_2 + \left(\frac{r+\delta_k}{\alpha_x\,\chi\,N_0}\right)\overline{m}_x\right)\left(\left(\frac{r+\delta_k}{\alpha_x\,\chi\,N_0}\right)m_x - \tilde{\omega}_1\right)^{-1}. \tag{A25}$$

From (6) and (A23) we have

$$a = \varphi(z) \equiv \bar{k} + \frac{\bar{l}\,R}{r}. \tag{A26}$$

It is straightforward to check that all the variables can be expressed as functions of z and X at any point in time as follows: r and w by (A2) $\rightarrow$ $\bar{k}$ by (A24) $\rightarrow$ K_a by (A18) $\rightarrow$ K_i and K_x by (A21) $\rightarrow$ N_i, N_x, and N_a by (A1) $\rightarrow$ $\hat{y}$ by (A14) $\rightarrow$ $\bar{l}$ by (18) $\rightarrow$ R by (A13) $\rightarrow$ p_L by (A23) $\rightarrow$ a by (A25) $\rightarrow$ L_a, L_x and l_h by (A11) $\rightarrow$ p_a by (A5) $\rightarrow$ F_i by (1) $\rightarrow$ F_a by (3) $\rightarrow$ F_x by (7) $\rightarrow$ p_x by (8) $\rightarrow$ c_i, c_a, c_x, and s by (13). From this procedure, (18) and (11), we have

$$\dot{a} = \Lambda_0(z) \equiv s - a, \tag{A27}$$

$$\dot{X} = \Omega(z, X) \equiv \phi_0 X \left(1 - \frac{X}{\phi}\right) - F_x. \tag{A28}$$

Taking derivatives of (A26) with respect to t yields

$$\dot{a} = \frac{d\varphi}{dz}\dot{z}. \tag{A29}$$

Equal (A27) and (A29)

$$\dot{z} = \Lambda(z) \equiv \Lambda_0 \left(\frac{d\varphi}{dz}\right)^{-1}. \tag{A30}$$

From (A28) and (A30), we determine the motion of z and X. We thus proved the lemma.